

57. Cost of Transporting Goods A trucking company transports goods between Chicago and New York, a distance of 960 miles. The company's policy is to charge, for each pound, \$0.50 per mile for the first 100 miles, \$0.40 per mile for the next 300 miles, \$0.25 per mile for the next 400 miles, and no charge for the remaining 160 miles.

- Graph the relationship between the cost of transportation in dollars and mileage over the entire 960-mile route.
- Find the cost as a function of mileage for hauls between 100 and 400 miles from Chicago.
- Find the cost as a function of mileage for hauls between 400 and 800 miles from Chicago.

$0 \leq x \leq 100$ \$0.5/mile $x = 40 \text{ m}$ $\$0.5(40)$ <u>\$20</u>	$100 \leq x \leq 400$ \$0.4/mile $x = 250 \text{ m}$ $0.5(100)$ $+$ $0.4(250 - 100)$	$400 \leq x \leq 800$ 0.25/mile $x = 500 \text{ m}$ $0.5(100)$ $+$ $0.4(300)$ $+$ $0.25(500 - 400)$	$800 \leq x \leq 960$ \$0 $x = 900 \text{ miles}$ $0.5(100)$ $+$ $0.4(300)$ $+$ $0.25(400)$ $+$ $0(900 - 800)$
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$$C(x) = \begin{cases} 0.5x & ; 0 \leq x \leq 100 \\ 0.5(100) + (x - 100) \cdot 0.4 & ; 100 < x \leq 400 \\ 0.5(100) + 0.4(300) + (x - 400) \cdot 0.25 & ; 400 < x \leq 800 \\ 0.5(100) + 0.4(300) + 0.25(400) + 0 & ; 800 < x \leq 960 \end{cases}$$

3.3p1 Quadratic

□ The quadratic function

$$f(x) = ax^2 + bx + c$$

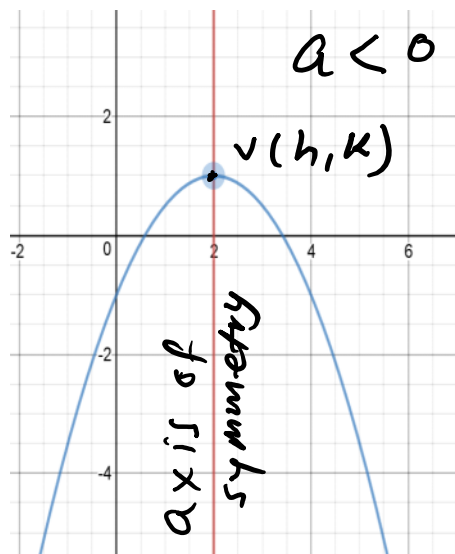
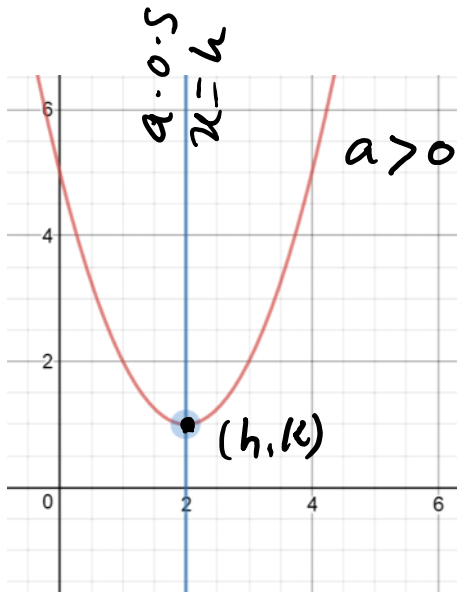
(general form)

$$f(x) = a(x-h)^2 + k$$

(standard form
vertex form)

$$\text{vertex: } (h, k)$$

$$\text{axis of symmetry: } x = h$$



y-int: $x = 0 \rightarrow y = c$ (0, c)

x-int: $y = 0 \rightarrow ax^2 + bx + c = 0$

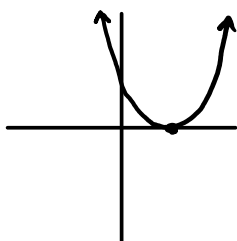
Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

the discriminant
of quadratic

The discriminant of the quadratic

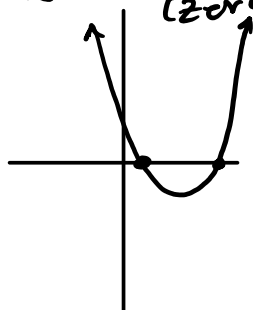
① $b^2 - 4ac = 0$

$x = -\frac{b}{2a}$ 1 real x-int



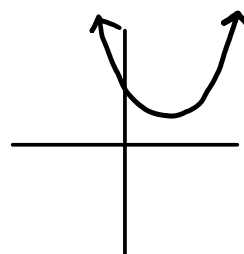
② $b^2 - 4ac > 0$

2 real x-int (zeros)



③ $b^2 - 4ac < 0$

2 imaginary zeros



3.3p2 Completing the Square

□ "Completing the square" to convert general to standard form:

$$y = ax^2 + bx + c \xrightarrow[\text{by CTS}]{\text{goal}} y = a(x-h)^2 + k$$

$$(A-B)^2 = A^2 - 2A \cdot B + B^2$$

$$y = x^2 - 6x + 5 \longrightarrow y = a(x-h)^2 + k$$

step 1: $y = 1(x^2 - 6x + ?) + 5$

step 2: $\frac{\text{middle}}{2} = \frac{6}{2} = (3)^2 = \boxed{9}$

step 3: $y = 1(x^2 - 6x + 9) + 5 - 9$

step 4: $y = 1(x-3)^2 - 4$

$$y = a(x-h)^2 + k$$

$$v(h, k) = (3, -4)$$

$$a.o.s: \boxed{x=3}$$

You may use any method (1 or 2) for x-intercepts!

$$y\text{-int: } (0, 5)$$

Now find x-int:

method 1. a-formula

$$1x^2 - 6x + 5 = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=1; b=-6; c=5$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{+6 \pm \sqrt{36 - 20}}{2} = \frac{6 \pm \sqrt{16}}{2}$$

$$x = \frac{6 \pm 4}{2} \begin{cases} \rightarrow x = \frac{10}{2} = 5 \quad (5, 0) \\ \rightarrow x = \frac{2}{2} = 1 \quad (1, 0) \end{cases}$$

Method 2:

Start w/ std. form

$$1(x-3)^2 - 4 = 0$$

$$\sqrt{(x-3)^2} = \sqrt{4}$$

$$x-3 = \pm 2$$

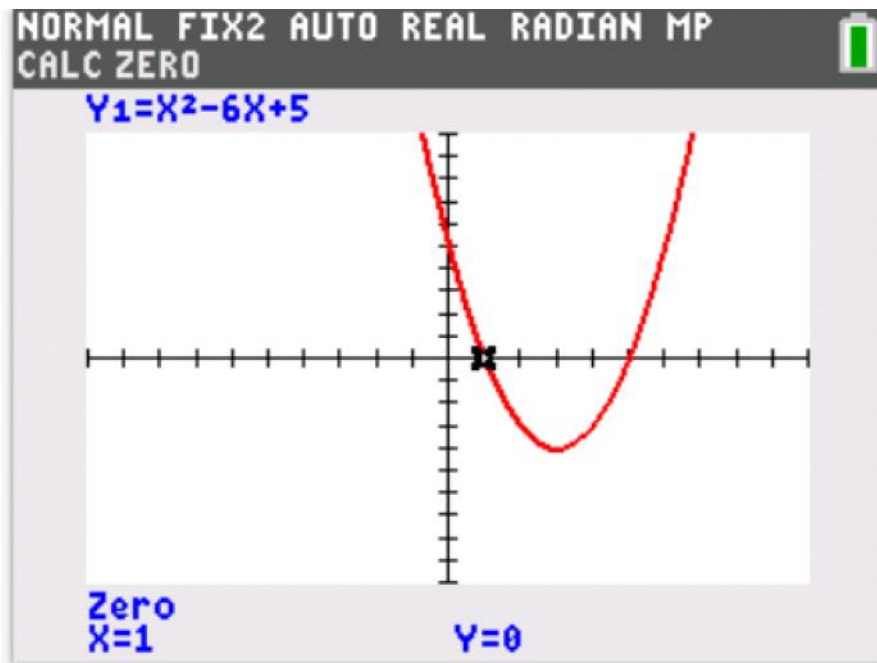
$$\begin{aligned} x &= -2 + 3 \\ x &= +2 + 3 \\ &= 5 \end{aligned}$$

$$\boxed{(5, 0)}$$

$$\begin{array}{r} x-3 = \pm 2 \\ +3 \quad +3 \\ \hline \end{array}$$

$x = +2 + 3$
5
 $(5, 0)$

3.3p3Both graphs



3.5p4(completing the square)

Steps to complete the square from
 $y = ax^2 + bx + c$ to $y = a(x-h)^2 + k$

1. separate the "c"

$$y = ax^2 + bx + c$$

2. Factor out "a"

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

3. Divide $\frac{b}{a}$ by 2 = $\frac{b}{2a}$

$$\text{square it} = \left(\frac{b}{2a}\right)^2$$

add it

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c$$

4. Subtract $a \times \frac{b^2}{4a^2}$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{ab^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a} + \frac{b^2}{4a^2}\right) + \left(c - \frac{b^2}{4a}\right)$$

5.

$$y = a\left(x + \frac{b}{2a}\right)^2 + k$$

Quadratic formula proof(NEW)

The Quadratic Formula:

x -intercept of $y = ax^2 + bx + c$

$$y = 0$$

$$\text{so, } 0 = ax^2 + bx + c$$

$$0 = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$0 = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$$

$$0 = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

$$\frac{b^2}{4a} - c = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2}{4a^2} - \frac{c}{a} = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$- \frac{b}{2a} \quad - \frac{b}{2a}$$

$$\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = x$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try:

$$y = -2x^2 + 4x - 8$$

$$y = (-2x^2 + 4x) - 8$$

$$y = -2(x^2 - 2x) - 8$$

$$\frac{2}{2} = (1)^2 = \boxed{1}$$

$$y = -2(x^2 - 2x + 1) - 8 + 2$$

$$y = -2(x - 1)^2 - 6$$

 x -int:

$$0 = -2(x - 1)^2 - 6$$

$$6 = -2(x - 1)^2$$

$$\sqrt{-3} = \sqrt{(x - 1)^2}$$

↓
imaginary

No x -int

 x -int:

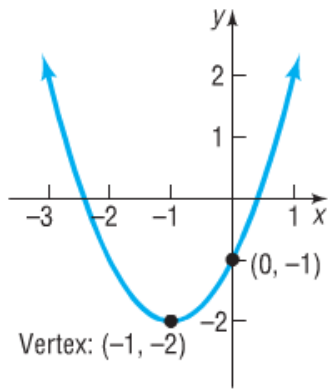
$$-2x^2 + 4x - 8 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(-8)}}{2(-2)}$$

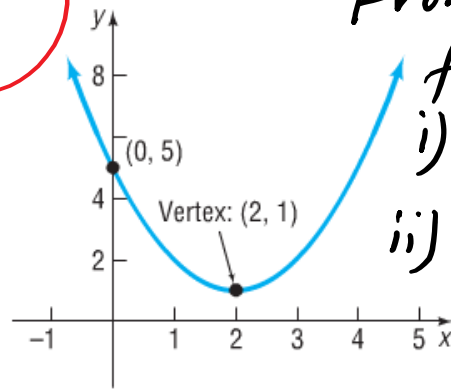
$$x = \frac{-4 \pm \sqrt{16 - 64}}{-4} \rightarrow \text{imaginary}$$

No x -int

49.

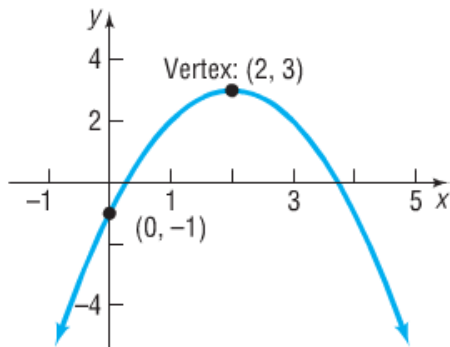


50.



From graph find
i) standard form
ii) general form

52.



50) given, $V(h, k)$
 $(2, 1)$
a pt: $(x, y) = (0, 5)$

i) Std. form:

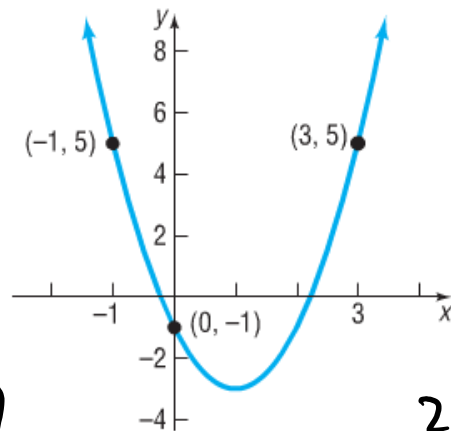
$$y = 1(x - 2)^2 + 1$$

$$y = 1(x^2 - 4x + 4) + 1$$

$$y = x^2 - 4x + 4 + 1$$

ii) $y = x^2 - 4x + 5$

53.



$$y = a(x - h)^2 + k$$

$$5 = a(0 - 2)^2 + 1$$

$$5 = a(4) + 1$$

$$5 = 4a + 1$$

$$-1 \quad -1$$

$$4 = 4a$$

$$1 = a$$

□ The short-cut method to get vertex
(without completing the square - use for applications)

given $y = ax^2 + bx + c$

$$(i) \rightarrow h = -\frac{b}{2a}$$

$$k = y\left(-\frac{b}{2a}\right)$$

example: $y = -2x^2 + 6x + 5$
 $a = -2; b = 6; c = 5$

$$h = -\frac{b}{2a} = -\frac{6}{2(-2)} = -\frac{6}{-4} = \left(\frac{3}{2}\right)$$

$$\boxed{V\left(\frac{3}{2}, \frac{19}{2}\right)}$$

$$k = y\left(\frac{3}{2}\right) = -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) + 5$$

$$= -2\left(\frac{9}{4}\right) + 9 + 5$$

$$= -\frac{18}{4} + 14$$

$$= -\frac{9}{2} + 14 = \boxed{\frac{19}{2}}$$

93. Business The monthly revenue R achieved by selling x wristwatches is figured to be $R(x) = 75x - 0.2x^2$. The monthly cost C of selling x wristwatches is $C(x) = 32x + 1750$.

- How many wristwatches must the firm sell to maximize revenue? What is the maximum revenue?
- Profit is given as $P(x) = R(x) - C(x)$. What is the profit function?
- How many wristwatches must the firm sell to maximize profit? What is the maximum profit?
- Provide a reasonable explanation as to why the answers found in parts (a) and (c) differ. Explain why a quadratic function is a reasonable model for revenue.

$$R(x) = 75x - 0.2x^2$$

$$C(x) = 32x + 1750$$

In application problems use $h = -\frac{b}{2a}$
 $K = c - \frac{b^2}{4a}$

Ind. var: $x = \text{Num. of watches}$

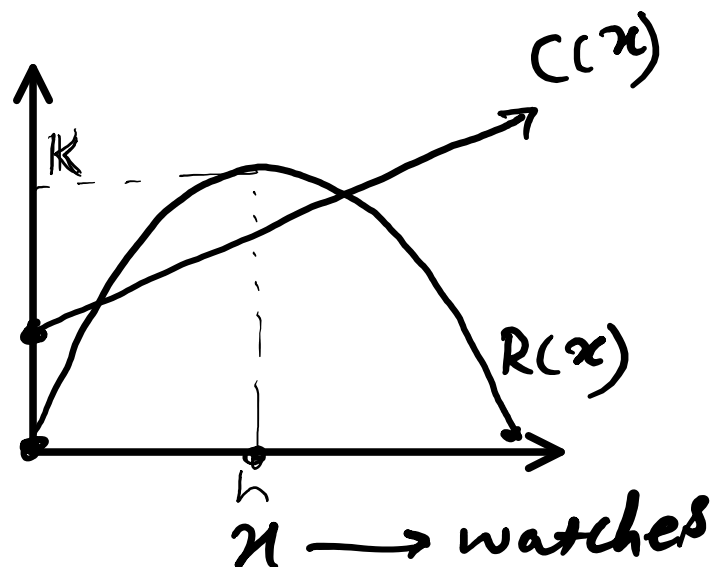
Dep. var: $R(x) = \text{Rev. \$}$

$C(x) = \text{Cost \$}$

$$R(x) = -0.2x^2 + 75x + 0$$

$$h = -\frac{b}{2a} = -\frac{75}{2(-0.2)} \approx 188 \text{ watches}$$

a) selling 188 watches maximizes Rev.



b) Profit function $P(x) = R(x) - C(x)$

$$= 75x - 0.2x^2 - (32x + 1750)$$

$$= -0.2x^2 + 75x - 32x - 1750$$

$$P(x) = -0.2x^2 + 43x - 1750$$

at breakeven: $R(x) = C(x)$, find x - the breakeven number of wristwatches.

$$75x - 0.2x^2 = 32x + 1750$$

on
calc

$$y_1 = 75x - 0.2x^2$$

$$y_2 = 32x + 1750$$

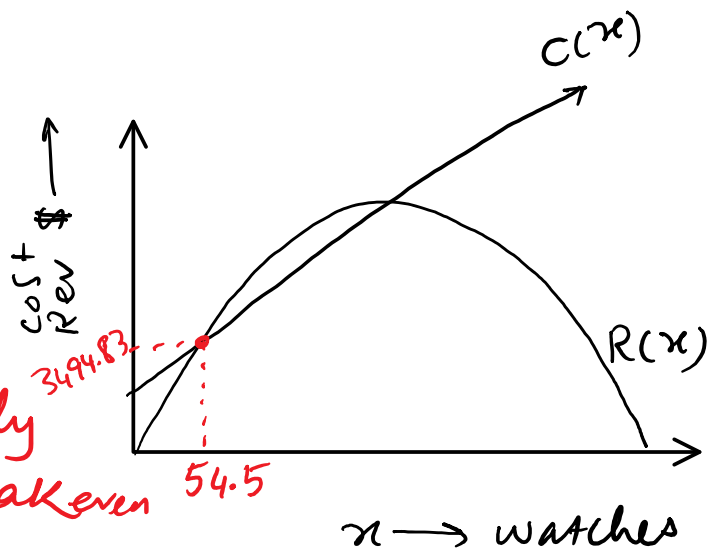
use 2nd calc INTERSECT to find the 1st point of intersection

$$x_{\min} = 0$$

$$x_{\max} = 400$$

$$y_{\min} = 0$$

$$y_{\max} = 10,000$$



Needs approximately
55 watches to breakeven
Breakeven Cost and Revenue
is \$ 3494.83

3.3#90(NEW)

- 90. Maximizing Revenue** The John Deere company has found that the revenue, in dollars, from sales of riding mowers is a function of the unit price p , in dollars, that it charges. If the revenue R is

$$R(p) = -\frac{1}{2}p^2 + 1900p$$

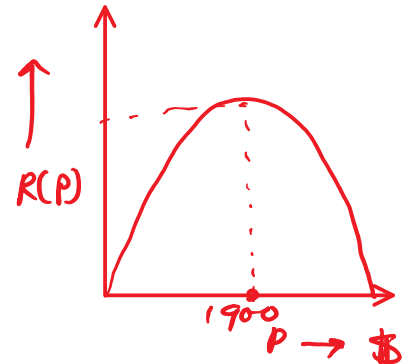
what unit price p should be charged to maximize revenue?
What is the maximum revenue?

$p \rightarrow$ Price / mower \$

$R(p) =$ Revenue \$

$$R(p) = -\frac{1}{2}p^2 + 1900p + 0$$

$$ax^2 + bx + c$$



Vertex (h, k) : $h = \frac{-b}{2a} = \frac{-1900}{2(-\frac{1}{2})} = \underline{\underline{1900}}$

They should charge \$ 1900/mower to maximize revenue

$$k = R(1900) = -\frac{1}{2}(1900)^2 + 1900(1900) =$$

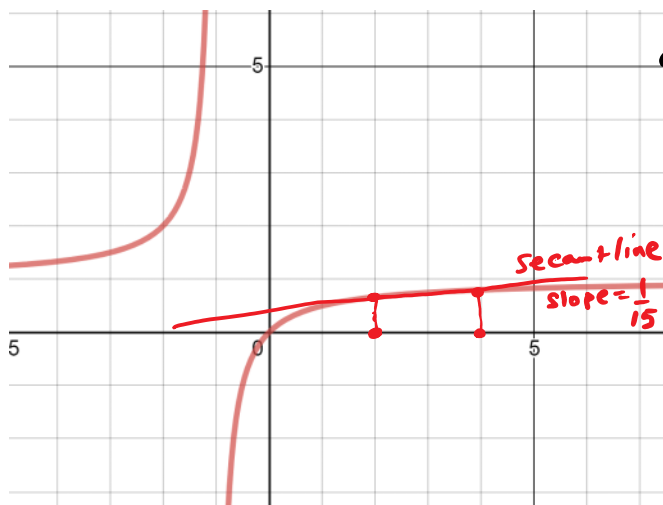
$\$1,805,000$ max revenue

DQ

Consider the function $f(x) = \frac{x}{x+1}$

No partial credit. 10 mins. Write response on "chat" box

- Find the reduced expression for its difference-quotient
- Find the reduced fraction for its Average Rate of Change in $[2, 4]$



$$f(x) = \frac{x}{x+1}$$

$$a) DQ: \frac{\left[\frac{x+h}{x+h+1} \right] - \left[\frac{x}{x+1} \right]}{\frac{h}{1}}$$

$$\frac{(x+1)(x+h) - x(x+h+1)}{(x+h+1)(x+1)}$$

$$\frac{x^2 + xh + x + h - x^2 - xh - x}{(x+h+1)(x+1)}$$

$$= \frac{\cancel{h}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{1}{(x+h+1)(x+1)}}$$

Keep bottom factored.

$$b) ARC: \frac{f(4) - f(2)}{4 - 2} = \frac{\left[\frac{4}{5} \right] - \left[\frac{2}{3} \right]}{\frac{2}{1}} = \frac{\frac{12-10}{15}}{\frac{2}{1}} = \frac{2}{15} \cdot \frac{1}{2} = \boxed{\frac{1}{15}}$$